



EASTERN UNIVERSITY, SRI LANKA EXTERNAL DEGREE EXAMINATION IN SCIENCE SECOND YEAR FIRST SEMESTER - 2003/2004, 2004/2005(July/August, 2008) DEPARTMENT OF MATHEMATICS EXTMT 207 - NUMERICAL ANALYSIS (PROPER & REPEAT)

Answer all Questions Calculators are provided Time: Two hours

- Q1. (a) By writing any real number p in a normalized decimal form, explain the terms "chopping" and "rounding".
 - (b) In a floating point number system, prove that

 $|\text{relative round-off error}| \leq \left\{ \begin{array}{ll} \beta^{1-t}, & \text{for chopping}; \\ \frac{1}{2} \ \beta^{1-t}, & \text{for rounding}, \end{array} \right.$

where β and t denote the base number and a maximum number of decimal digits, respectively.

- (c) If three approximated values of the number $\frac{1}{3}$ are 0.30, 0.33 and 0.34, which of these is the best approximation?
- Q2. (a) Let x = g(x) be an arrangement of the equation f(x) = 0, which has root α in the interval *I*. If g'(x) exists and continuous in *I* satisfying

$$|g'(x)| \le h < 1, \ \forall x \in I,$$

prove that, for any given x_0 , the sequence $\{x_r\}, r = 0, 1, 2, \dots$, defined by

$$x_{r+1} = g(x_r)$$

converges to α and such α is unique.

Hence, find the condition for the convergence of Newton-Raphson method.

- (b) Find a root of the equation $x^3 x 1 = 0$ correct to four decimal places us Newton-Raphson and Secant methods. Compare the result you have obtained
- Q3. (a) Write down the Lagrange's interpolation formula and show that such interp lation formula p of $f : [a, b] \longrightarrow \mathbb{R}$ satisfying $p(x_i) = f(x_i), i = 0, 1, ...,$ where x_i 's are distinct in [a, b], always exists and is unique.
 - (b) Prove that the error in Lagrange's interpolation has the form

$$\frac{(x-x_0)(x-x_1)(x-x_2)\dots(x-x_n)}{(n+1)!}f^{n+1}(\xi), \ \xi \in (a,b).$$

(c) Find the Lagrange's interpolating polynomial of degree 2 approximating the function $y = \ln x$ using the following tabular values. Hence, find the value $\ln(2.7)$ and determine the error using part (b).

x	2.00000	2.50000	3.00000
$\ln x$	0.69315	0.91629	1.09861

Q4. (a) Write down the formula for the integral

$$\int_{x_0}^{x_1} f(x) \ dx$$

and its error term, which represent the trapezoidal rule in the interval $[x_0, x_1]$ Hence, derive the composite form of the trapezoidal rule and its error ten over the interval [a, b].

(b) Use composite trapezoidal to evaluate an approximate value of

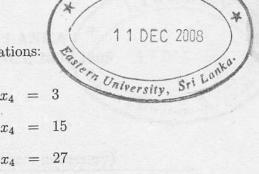
$$\int_0^1 \frac{1}{1+x} \, dx$$

correct to three decimal places using the following table:

x	0.0000	0.2500	0.5000	0.7500	1.0000
y	1.0000	0.8000	0.6667	0.5714	0.5000

Estimate truncation and round-off errors.

(c) Consider the following system of linear equations:



 $10x_1 - 2x_2 - x_3 - x_4 = 3$ -2x_1 + 10x_2 - x_3 - x_4 = 15 -x_1 - x_2 + 10x_3 - 2x_4 = 27 -x_1 - x_2 - 2x_3 + 10x_4 = -9.

Use Gauss-Seidel method to carry out 3 iterations for x_1, x_2, x_3 and x_4 correct to 4 decimal places.